NAG Toolbox for MATLAB

s15ad

1 Purpose

s15ad returns the value of the complementary error function, $\mathrm{erfc}(x)$, via the function name.

2 Syntax

```
[result, ifail] = s15ad(x)
```

3 Description

s15ad calculates an approximate value for the complement of the error function

$$\mathrm{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^\infty e^{-t^2}\, dt = 1 - \mathrm{erf}(x).$$

Let $\hat{x}$ be the root of the equation $\mathrm{erfc}(x) - \mathrm{erf}(x) = 0$ (then $\hat{x} \approx 0.46875$). For $|x| \leq \hat{x}$ the value of $\mathrm{erfc}(x)$ is based on the following rational Chebyshev expansion for $\mathrm{erf}(x)$:

$$\mathrm{erf}(x) \approx x R_{\ell,m}\left(x^2\right),$$

where $R_{\ell,m}$ denotes a rational function of degree ℓ in the numerator and m in the denominator.

For $|x| > \hat{x}$ the value of $\mathrm{erfc}(x)$ is based on a rational Chebyshev expansion for $\mathrm{erfc}(x)$: for $\hat{x} < |x| \leq 4$ the value is based on the expansion

$$\mathrm{erfc}(x) \approx e^{x^2} R_{\ell,m}(x);$$

and for $|x| > 4$ it is based on the expansion

$$\mathrm{erfc}(x) \approx \frac{e^{x^2}}{x}\left(\frac{1}{\sqrt{\pi}} + \frac{1}{x^2} R_{\ell,m}\left(1/x^2\right)\right).$$

For each expansion, the specific values of ℓ and m are selected to be minimal such that the maximum relative error in the expansion is of the order 10^{-d}, where d is the maximum number of decimal digits that can be accurately represented for the particular implementation (see x02be).

For $|x| \geq x_{\mathrm{hi}}$ there is a danger of setting underflow in $\mathrm{erfc}(x)$. For $x \geq x_{\mathrm{hi}}$, s15ad returns $\mathrm{erfc}(x) = 0$; for $x \leq -x_{\mathrm{hi}}$ it returns $\mathrm{erfc}(x) = 2$.

4 References

Abramowitz M and Stegun I A 1972 *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

Cody W J 1969 Rational Chebyshev Approximations for the Error Function *Math.Comp.* **23** 631–637

5 Parameters

5.1 Compulsory Input Parameters

1: **x – double scalar**

The argument x of the function.

5.2 Optional Input Parameters

None.

5.3 Input Parameters Omitted from the MATLAB Interface

None.

5.4 Output Parameters

1: **result – double scalar**

The result of the function.

2: **ifail – int32 scalar**

0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

There are no failure exits from s15ad. The parameter **ifail** has been included for consistency with other functions in this chapter.

None.

7 Accuracy

If δ and ϵ are relative errors in the argument and result, respectively, then in principle

$$|\epsilon| \simeq \left| \frac{2xe^{-x^2}}{\sqrt{\pi}\,\mathrm{erfc}(x)} \delta \right|.$$

That is, the relative error in the argument, x, is amplified by a factor $\frac{2xe^{-x^2}}{\sqrt{\pi}\,\mathrm{erfc}(x)}$ in the result.

The behaviour of this factor is shown in Figure 1.

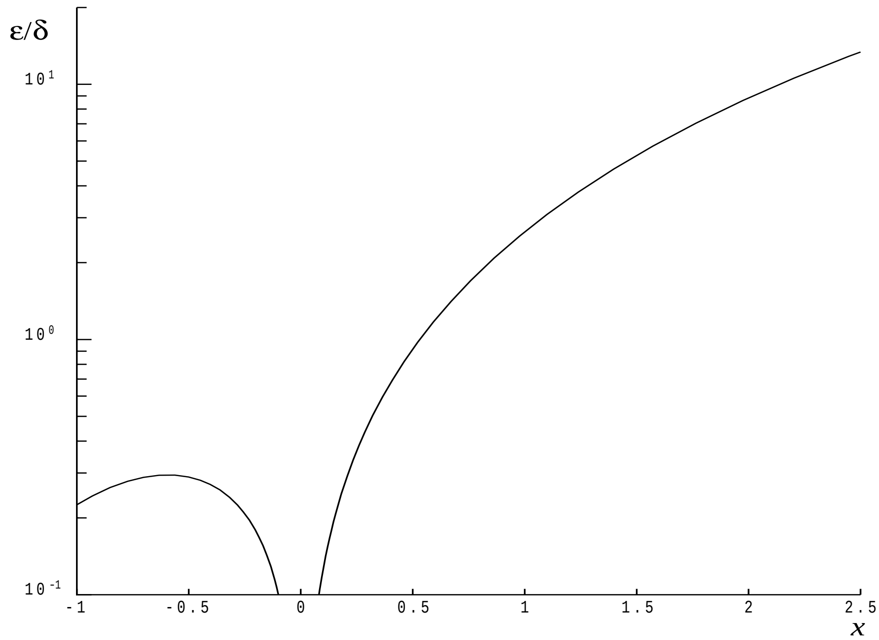

Figure 1

It should be noted that near $x = 0$ this factor behaves as $\frac{2x}{\sqrt{\pi}}$ and hence the accuracy is largely determined by the ***machine precision***. Also for large negative x, where the factor is $\sim \frac{xe^{-x^2}}{\sqrt{\pi}}$, accuracy is mainly limited by ***machine precision***. However, for large positive x, the factor becomes $\sim 2x^2$ and to an extent relative accuracy is necessarily lost. The absolute accuracy E is given by

$$E \simeq \frac{2xe^{-x^2}}{\sqrt{\pi}}\delta$$

so absolute accuracy is guaranteed for all x.

8 Further Comments

None.

9 Example

```
x = -10;
[result, ifail] = s15ad(x)
```

```
result =
     2
ifail =
           0
```